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ABSTRACT

This study investigated the actual learning processes of two preservice teachers as they explored geometry problems with dynamic geometry software and the effects of using the software on developing their mathematical reasoning and proof abilities. A constructivist teaching experiment was designed and implemented for the investigation. Through participating in the study, the subjects developed a new learning style--exploring problem situations through a learning process characterized by initial conjecture, investigation--more thoughtful conjecture, verification (or proof)--proof (or verification). They changed their conceptions of mathematics and mathematics teaching as well. The Geometer's Sketchpad (a dynamic geometry software package) was found to be an excellent teaching and learning tool that can enhance students' mathematical reasoning and proof abilities. (Author)

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DEVELOPING PRESERVICE TEACHERS' MATHEMATICAL REASONING AND PROOF ABILITIES IN THE GEOMETER'S SKETCHPAD ENVIRONMENT

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This study investigated the actual learning processes of two preservice teachers as they explored geometry problems with dynamic geometry software and the effects of using the software on developing their mathematical reasoning and proof abilities. A constructivist teaching experiment was designed and implemented for the investigation. Through participating in the study, the subjects developed a new learning style—exploring problem situations through a learning process characterized by initial conjecture – investigation – more thoughtful conjecture – verification (or proof) – proof (or verification). They changed their conceptions of mathematics and mathematics teaching as well. The Geometer's Sketchpad (a dynamic geometry software package) was found to be an excellent teaching and learning tool that can enhance students' mathematical reasoning and proof abilities.

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A research study was conducted in the Fall semester 2001 within the course "Learning Mathematics with Technology" (MAE 3651) for secondary school mathematics preservice teachers at Florida International University. The goal of the course in which the subjects were enrolled during this study is to help the preservice teachers experience learning mathematics with technology, determine important content areas for the secondary school mathematics curriculum and develop competency in these content areas. Using dynamic geometry software to revisit high school geometry is a major focus of the course.

The Purpose of the Research

The purpose of the research was to investigate the actual learning processes of the preservice teachers as they explored geometry problems with dynamic geometry software and to find the effects of the use of dynamic geometry software on developing their mathematical reasoning and proof abilities.

Conceptual Framework

The conceptual framework of this study came out of the constructivist perspective that knowledge is not passively received from the teacher but actively constructed by the learners themselves, the van Hiele model of geometric thinking, and the research on mathematical reasoning and proof. According to van Hiele (1986), students progress in geometric thinking through a taxonomy of levels, and progress from one level to the next higher level is dependent on the nature of the instruction provided to students. The van Hiele levels are numbered differently in various sources. Based on the numbering system used by Battista & Clements (1995), the levels are: Level 1 - visual, Level 2 - descriptive / analytic, Level 3 - abstract / relational, Level 4 - formal deduc-

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tion, and Level 5 – rigor / metamathematical. de Villiers (1987) suggests that deductive reasoning in geometry first occurs at level 3 when the network of logical relations between properties of concepts is established. He claims that because students at levels 1 or 2 do not doubt the validity of their empirical observations, formal proof is meaningless to them - they see it as justifying the obvious (Battista & Clements, 1995). Other researchers (Senk, 1987; Van Dormolen, 1977) give similar suggestions.

Design and Procedure

A constructivist teaching experiment was designed and implemented for the investigation. During the teaching experiment, I attempted to provide opportunities for the students (preservice teachers, the same hereafter with only a few exceptions) to learn by judiciously selecting tasks, posing questions, and encouraging active explorations. The study lasted for ten weeks when the course concentrated on geometry topics using the Geometer's Sketchpad (GSP), a dynamic geometry software package. Since the students' detailed changes and growth in their learning processes were the focus of the investigation, two students were selected from the MAE 3651 class to participate in the study. They did not join the big class during the ten weeks except for the first class session in which a pre-test was administered. Teaching interviews (two times a week with 75 minutes each time) were conducted for them, sometimes individually, and sometimes in a pair (for cooperative learning purposes). The time for the subjects to learn geometry and the problems for them to explore were basically the same as those for the big class. The difference between the two settings was that there was much more one-on-one interaction between me as the instructor and the subjects in the teaching interviews. The two subjects were selected based on both their willingness of participation and their different thinking levels in the van Hiele model.

The study consisted of three phases. Phase 1 was the pre-test. Phase 2 consisted of nine-week teaching interviews. Phase 3 was the post-test.

The Pre-test

For the pre-test, I used 30 questions - 20 questions selected from Mayberry's (1981) instrument and 10 questions that I had devised - to assess the student's van Hiele levels of geometric thinking. All questions dealt with triangles and quadrilaterals. The 10 questions that I had devised looked more on whether a student can do simple proofs. The 10 questions and the criteria for scoring were developed with the same structure as that of Mayberry's instrument. From among the students who were interested in participating in the study, a male student (Fred) and a female student (Lisa) were chosen. It was found that Lisa was at level 3 and Fred was at level 2 in the van Hiele model.

The Teaching Interview

The teaching interviews for each of the two subjects explored the detailed processes in which the students' problem solving, mathematical reasoning, and proof abilities were enhanced. The subjects worked on the assigned activities using GSP as

a tool. I interacted with them by probing with questions, and giving hints only when they needed help.

The problem situations used for the interviews were designed based on the following considerations: 1) The tasks assigned to the subjects should "focus on important mathematics" (NCTM, 2000, p.15); 2) These tasks should be both challenging and consistent with the subjects' van Hiele levels of geometric thinking; 3) The sequence of tasks should help the subjects gradually progress from lower levels of geometric thinking to higher levels; 4) The tasks should emphasize the subjects' active hands-on activities with the dynamic geometry software to assist their mathematical thinking; and 5) The tasks should allow the subjects to generate feedback from which they can judge the efficacy of their methods of thinking. Nine sets of problems were designed and assigned to the subjects during the interview sessions. Solving these problems required the grasp of important concepts and relationships related to the following three topics: Triangles, Quadrilaterals/Polygons, and Circles. Since a proof-oriented geometry course requires thinking at least at level 3 in the van Hiele hierarchy (Battista & Clements, 1995), the problems (especially the problems at the early stage of the interviews) given to Fred didn't require formal proofs but informal arguments, while all problems given to Lisa required both intensive investigations and formal proofs. The problems were sequenced so that the level of sophistication and difficulty of problem contexts increased gradually. Taking full advantage of the dynamic, multiple, linked representations given by GSP was encouraged in explorations of the problems.

The Post-test

At the end of the ten weeks, the subjects took a written examination (with the other students in class) and an additional post-test. The post-test given to Fred was the same as the pre-test except for three items, which were new activities for him and went beyond any of the tasks he encountered during the interview sessions. The post-test given to Lisa consisted of 10 proof-oriented questions or problems.

Data Collection and Analysis

Each teaching interview was videotaped. All significant parts of the videotapes were transcribed. Careful notes were taken for all interview sessions and related after-session discussions. The answer sheets of the pre-test and the post-test, the computer files on the disks, as well as the written work (assignments and tests/exams) and comments completed by the students in each interview session, were also collected for analysis

A constant comparison approach (Glaser & Strauss, 1967) was used in this study for the data analysis. This method of analysis is inductive - it moves from data to tentative theory, to new data, to refined theory. The transcripts of the videotapes and other data were analyzed during interview sessions and after all sessions were completed. These analyses took the form of editing data with commentaries. The commentaries

depicted the story lines of the corresponding students' progression through the sessions and revealed the critical developmental points of the students.

Results

A New Learning Style

In the teaching interviews, the subjects were encouraged to explore each problem situation through a learning process characterized by initial conjecture - investigation – more thoughtful conjecture – verification (or proof) – proof (or verification). This learning process effectively facilitated the subjects' development of mathematical reasoning and proof abilities.

During the third week of Phase 2, the subjects were assigned to solve the problem below:

> A road is proposed that will connect two towns A and B on opposite sides of a river. The road will cross the river in a bridge that is perpendicular to the riverbanks. Where should the bridge be placed so as to minimize the total length of the road?

First, the subjects were encouraged to give their initial conjectures. Lisa used her knowledge from a previous problem in which the river was simply a line (river width ignored). In that case, the shortest path was the straight line segment from point A to point B. So, she automatically assumed the same approach for this problem. She drew a straight line connecting points A and B. This time, however, since the river had a width, the line intersected the river at two points. She decided to find the midpoint of the segment determined by these two intersection points. She then went on to conjecture that it was the segment through this midpoint perpendicular to the riverbanks that would give the shortest path from A to B. Fred had a different conjecture. He believed that the bridge on the shortest path should lie on the line passing through the midpoint of segment AB and perpendicular to the riverbanks.

The subjects then tested their conjectures through experimentation with GSP. Using the dynamic feature of GSP, Lisa was able to construct an arbitrary path and move it along the river until she found the shortest path (path AMNB in Figure 1) by using the Measure tool. This path disproved her initial conjecture. Fred "located" the shortest path in a similar way. His conjecture was also found to be incorrect.

However, the shortest path that was "located" was not the real path that must be constructed. The subjects were led to carefully observe the "located" shortest path, and try to find its characteristics. Through observation and checking by doing necessary measurements, Lisa found that the most important characteristic of the path was segment AM // segment NB. Because of this, if AM was moved down by the width of the river, then AM and NB lied on the same line. In addition, point N that located the bridge on the shortest path was the intersection point of this line and the bottom riverbank. This analysis revealed an idea to construct the shortest path - Translate point A by vector XX' (Segment XX' in Figure 1 represents the river width.) to locate point

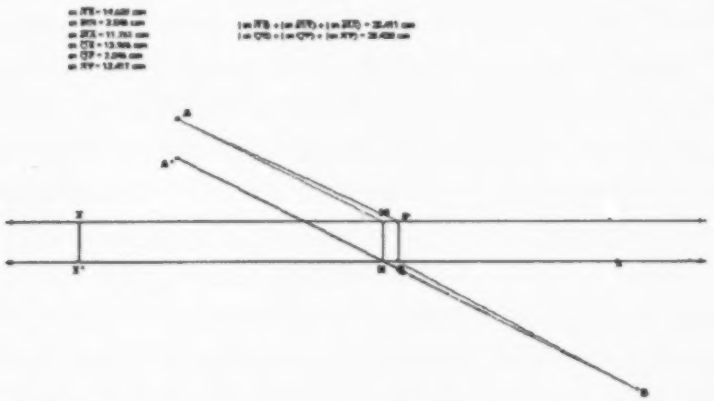

Figure 1

A', connect points A' and B using a segment, and construct the intersection point of segment A'B and the bottom riverbank. Then it was easy to complete the rest of the steps of the construction. Fred had difficulty understanding the process. I arranged the interview session so that Lisa and Fred worked together as a group. Lisa was happy to answer questions that Fred asked. Both subjects completed the construction of the shortest path - in other words, made a thoughtful conjecture.

(In order to save space by not introducing another figure, let's now consider path AMNB in Figure 1 to be the constructed shortest path.) To verify whether the shortest path was constructed correctly, both subjects created an arbitrary path APQB where Q was a free point on riverbank k and segment PQ represented the bridge (see Figure 1). By continuously dragging point Q and comparing the length of path APQB with that of path AMNB, both subjects observed that path APQB was always longer than path AMNB. Therefore, AMNB was indeed the shortest path.

Since verification itself is not a proof, the problem was not solved yet. Lisa continued her GSP exploration to construct a proof. She began dragging different points around to see how it would affect the distance of the paths, hoping that she would find a clue for proving the conjecture. As she was doing this, she came across exactly what she needed. She found that if she dragged one of the riverbanks close enough to the other so as to make the river width approach 0, a triangle was formed with sides AP, PB, and AB. This meant that when the river width was 0, the constructed path became side AB, and the arbitrary path became sides AP and PB. By the Triangle Inequality Theorem, AB AP + PB, confirming that AMNB is the shortest path. Since the river did have a width, the situation was different. However, the only difference was the river width, and the width was included in both paths. When the river width was subtracted from both paths, the situation would be similar to that when the river width

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was 0. To remove the river width from both paths, the only thing needed to do was to translate segment AM by vector XX' (see Figure 1). After the translation, the parallel segments AM and NB would become one segment, and the Triangle Inequality Theorem would be used to complete the proof.

An Excellent Teaching and Learning Tool That Can Enhance Students' Mathematical Reasoning and Proof Abilities

GSP has been widely used by teachers, mathematics educators, and students at both school and college levels as an effective teaching and learning tool. However, most of the people use GSP dynamics to help discover properties and relationships, make and test conjectures, and construct geometric objects. In this study, I not only emphasized these aspects of the use of GSP, but also went one step further. I also emphasized that after conjectures were made and tested, the subjects continued GSP explorations to come up with insight for reasoning and proofs. By analyzing the data that I collected in the study, my finding supported that of Battista & Clements (1995) - Sketchpad explorations can not only encourage students to make conjectures, they can foster insight for constructing proofs.

A good example of this aspect has been seen in the Shortest Path problem: Dragging one of the riverbanks close enough to the other and observing the resulting situation fostered insight for proof. Another example in which GSP helped the subjects both make and prove their conjecture was the problem solving process below:

> ABC is an arbitrary triangle. Points D, E, and F are respectively on sides BC, CA, and AB. BD= (1/3)BC, CE = (1/3)CA, and AF= (1/3)AB. PQR is formed by the construction of line segments AD, BE, and CF. What is the relationship between PQR and ABC? (see Figure 2)

When asked to make an initial conjecture, Lisa observed the figure for a while, and then said, "If you ask for the area relationship, it's 1/3, ... no, 1/9, ... no, I am not sure. Less than 1/3." Fred chose 1/9 without being sure either. Through GSP investigations, both subjects found that area(PQR) = (1/7)area(ABC). Because this problem was quite challenging, the task to prove the finding/conjecture was only assigned to Lisa. She spent quite a while thinking about a proof, but all efforts "led to no where" (her own words). In this case, I asked her to continue her investigation with GSP. Through further work with GSP measurements, she found that Area(BDP) = Area(CEQ) = Area(AFR) = (1/3)Area(PQR) = (1/21)Area(ABC); Area(Quadrilateral DCQP) = Area(Quadrilateral EARQ) = Area(Quadrilateral FBPR) = (5/3)Area(PQR) = (5/21)Area(ABC); BP : PQ : QE = 3 : 3 : 1; CQ : QR : RF = 3 : 3 : 1; and AR : RP : PD = 3 : 3 : 1. She was very interested in these new findings. I suggested that instead of proving the original conjecture directly, she might try to prove a new finding first - for instance, prove the relationship between the smallest triangles (such as BDP) and ABC. She agreed, and continued her active thinking. With minimal help from me (I asked a question: Let x be the area of BDP and y be the area of CEP . What are the areas of DCP and EAP?), she fi nally came up with a proof similar to the one shown in Figure 2:

Let x be Area(ΔBDP), then Area(ΔCDP)=2x
(because of same height and double base);
Let y be Area(ΔCEP), then Area(ΔAEP)=2y
(same reason). Hence, we have
x + 2x + y = Area(ΔBCE) = (1/3)Area(ΔABC) ... (1)
2x + y + 2y = Area(ΔADC) = (2/3)Area(ΔABC) ... (2)
By simple calculations on (1) and (2), we have
x = (1/21)Area(ΔABC). Using similar method,
we can get Area(ΔCEQ) = x = (1/21)Area(ΔABC),
and Area(ΔAFR) = x = (1/21)Area(ΔABC).
Therefore, Area(ΔPQR) = Area(ΔADC) - Area(ΔARC)
- Area(Quadrilateral DCQP) = (2/3)Area(ΔABC) -
[Area(ΔAFC) - x] - [Area(ΔBCE) - x - x] =
(2/3)Area(ΔABC) - [(1/3)Area(ΔABC) - x]
- [(1/3)Area(ΔABC) - 2x] = x + 2x = 3x
= 3 * (1/21) Area(ΔABC) = (1/7) Area(ΔABC).

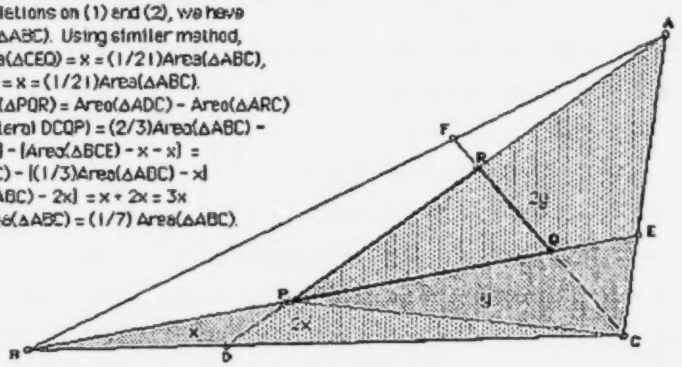

Figure 2

She deeply felt that GSP can foster insight for constructing proofs and hence is an excellent teaching and learning tool that can enhance students' mathematical reasoning and proof abilities.

Different Roles in Helping Students at Different Levels of Geometric Thinking

Research has demonstrated the effectiveness of using GSP in geometry teaching and learning (Dixon, 1997; Choi-koh, 1999). Based on the data collected in this study, I found that GSP plays different roles in helping students at different levels of geometric thinking. The following problem and the subjects' exploration processes provide an example.

> STREET PARKING. You are on the planning commission for Algebraville, and plans are being made for the downtown shopping district revitalization. The streets are 60 feet wide, and an allowance must be made for both on-street parking and two-way traffic. Fifteen feet of roadway is needed for each lane of traffic. Parking spaces are to be 16 feet long and 10 feet wide, including the lines. You job is to determine which method of parking – parallel or angle – will allow the most room for the parking of cars and still allow a two-way traffic flow. (You may design parking for one city block (0.1 mile) and use that design for the entire shopping district.)

Both Lisa and Fred gave the same initial conjecture: Angle parking would allow more cars to be parked than parallel parking. Then they began their investigations.

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Lisa used paper and pencil to draw diagrams to help thinking. She quickly determined how many cars could be parked in one city block in the parallel parking situation by saying, "The length of the block (528 ft) would be divided by 16 ft, the length of each parking space. This would allow 528/16 = 33 parking spaces on each side of the street, giving a total of 66 available parking spaces."

For angle parking, Lisa's investigation process was not so smooth as that for parallel parking. Her diagram is shown in Figure 3. She stated, "Using the Pythagorean Theorem to find x, we get $x = \sqrt{16^2 - 15^2} = \sqrt{31} \approx 5.567$ (feet). Now we are left with 528 – 5.567 = 522.433 (feet). If we divide this length by the width of the parking space, 522.433/10 = 52.24 (feet). Then, *at most* 52 cars would be able to park on each side of the road. Thus, giving 104 available parking spaces."

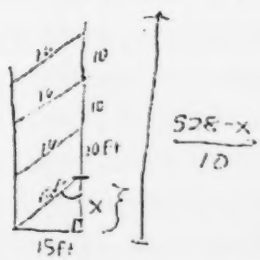

Figure 3

Figure 4

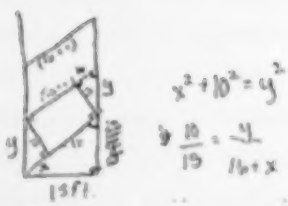

Figure 5

While indicating that it was good for her to understand the length of the curb space (10 ft in her thinking) was the key to determine how many cars could be parked, I had the following dialogue with her (I stands for the investigator and L stands for Lisa):

I: If a car is exactly 16 feet long and 10 feet wide, can the car be parked in your parking space? (I drew a rectangle on paper to represent a car.)

L: (after thinking for a while) No. The parking space is too small in comparison to the car.

I: What does this tell you?

L: My drawing is wrong.

I: Do you still think angle parking is better?

L: I don't know yet. Need more work.

I: How will you correct the mistake in your diagram?

L: The rectangle should be inside the parking space. (She immediately drew a new diagram shown in Figure 4.)

Even though the new diagram was far from accurate in terms of ratios between segments, the structure was correct. Lisa continued active thinking, and in about ten minutes, she came up with a system of two equations shown in Figure 5. Solving the system of equations would give the length of the curb space, and then the number of cars that could be parked would be easily calculated.

Lisa did not stop at this point, but instead constructed the drawings shown in Figures 4 and 5 in GSP and used GSP measurements and calculations to verify the solution she had obtained. This became her habit later – using GSP as a tool to verify her work after she had developed mathematical solutions or formal proofs.

Fred experienced a much more difficult time in formulating a mathematical solution for the angle-parking situation although he also correctly calculated the number of cars that could be parked in the parallel parking situation, In order to help Fred, I opened a pre-constructed GSP sketch for him (Figure 6).

Below is the dialogue between Fred and me:

I: Look at the current position of the car parking. Is this situation OK?

F: No. It blocks the traffic.

I: How will you change the situation so that the parking is acceptable with the regulations?

F: (Looked at the sketch seriously, and then dragged the point "Drag" down until the yellow rectangle only "touches" the traffic lane [see Figure 7].)

I: What do you observe?

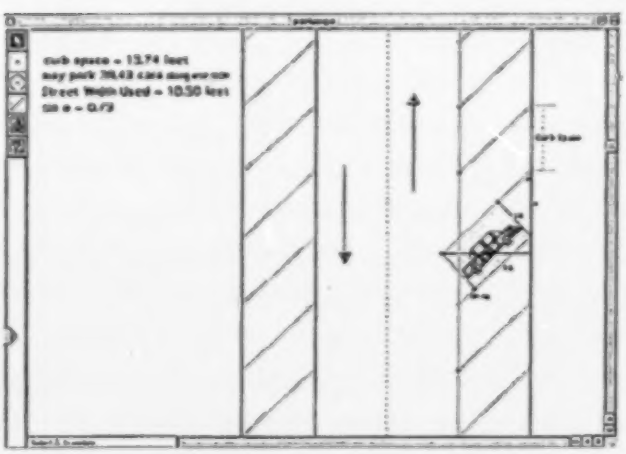

Figure 6

Figure 7

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F: (Thought for quite a while.) Now the traffic flows, but space is wasted a lot.

I: Did you see the numerical values displayed on the left-hand side?

F: Yes. Now the curb space is very long. Only 18 cars can be parked on one side of the road.

I: What is your conclusion?

F: Parallel parking is a better choice.

It seems that without GSP as an investigation tool, it would have been difficult for Fred to come up with the more thoughtful conjecture that parallel parking was a better choice. More help was provided to Fred when he tried to explain the new conjecture. This was not the case for Lisa. Sufficient GSP experience had helped her achieve a thinking level at which she was able to make thoughtful conjectures and even develop formal solutions or proofs (for many problems) with or without using GSP. In this case, GSP was more a verification tool than an investigation tool for her.

Improvement of the Subjects' van Hiele Levels of Geometric Thinking

At the beginning of the study, it was found via the pre-test that Lisa was able to perceive relationships between properties and understand logical implications and class inclusions. She was able to do some simple proofs using a two-column format, but not beyond that. For instance, she constantly used the Triangle Angle Sum Theorem, but did not know how to prove it. Therefore she was considered to be at van Hiele level 3 of geometric thinking. During the ten-week teaching interviews, she gradually progressed to level 4 at which she was able to self-initiate efforts to reason deductively and construct proofs. For the ten proof-oriented questions or problems in the post-test, Lisa correctly completed eight and partially completed two. The two problems that Lisa only partially completed had a certain degree of difficulty, and it was understandable that she was not able to complete them in limited time. Among the eight problems that she correctly completed, one was: *Two circles intersect at Points A and B. A line passing through A intersects the two circles respectively at C and D. A line passing through B intersects the two circles respectively at E and F. What is the relationship between lines (or segments) CE and DF? Can you prove your conjecture?* Lisa quickly conjectured that line segments CE and DF were parallel through visual observation. She also quickly verified her conjecture by dragging points on the circles to see if the lines would still look parallel, and they did. To prove the conjecture, however, was a challenge. It required the understanding of the Cyclic Quadrilateral Theorem and the theorems to prove two lines are parallel. Furthermore, the key for developing a proof was the flexibility to create cyclic quadrilaterals that did not exist explicitly in the first place. After active thinking, Lisa showed this flexibility by constructing segment AB to form two cyclic quadrilaterals. Then the rest of the proof became straightforward. It was evident that Lisa had progressed to solid van Hiele level 4 of geometric thinking.

Fred also demonstrated the development of his geometric thinking during the ten-week teaching interviews. His performance in the post-test showed that he had progressed from his original level (van Hiele level 2) to a thinking level at which he was able to identify relationships and categorize figures in a hierarchical order (van Hiele level 3). For example, he knew that a square is a rectangle, but a rectangle is not necessarily a square. Similarly, a rhombus is a parallelogram, but not vice-versa. He recognized that the set of all rectangles is bigger than the set of all squares. Furthermore, he was able to understand and sometimes even present logical arguments. For example, Fred had difficulty to do the deductive proof in the parking problem. However, when he discussed the problem with Lisa, he was able to understand Lisa's reasoning. After the discussion, he was able to explain why the parallel parking was better on his own.

Changes in the Subjects' Conceptions of Mathematics and Mathematics Teaching

The subjects' conceptions of mathematics and mathematics teaching significantly changed as a result of their experience in the interview sessions.

The interactive, dynamic, and innovative features of GSP allow students who might not be interested in mathematics to become active participants in exploring mathematics. Previously, teachers lectured and were the ones doing all the work on the board. Students had little determination so that they became unproductive and uninterested in the subject matter. However, with GSP, students now have the opportunity to get involved in the teaching and learning processes by making, verifying, and proving their own conjectures. Before participating in the teaching interviews, Fred didn't like geometry, and fell asleep in geometry classrooms. Now, he enjoys geometric learning. He indicated, "One of the reasons that I am so interested in this class is because we use computers. I am like a little kid and my attention is easily caught by the use of a computer."

Before the teaching interviews, Lisa agreed to the following statement, "To understand mathematics, students must solve many problems following examples provided." She explained that this was the way she had been taught mathematics her entire life. The teacher provides examples and the students imitate the procedures. She wouldn't know any other way of learning and teaching mathematics. However, after participating in the teaching interviews, she realized that students can understand mathematics, perhaps even better if they discover/explore solutions to mathematics problems themselves than if they simply mimic the teacher's examples. She indicated that, in the MAE 3651 class, the instructor hardly provided examples for students to follow. Rather, the students were given problem situations and had to find solutions themselves. This teaching method got all students involved and interested. In the process of finding a solution, students learned why the solution worked. Thus, they understood the related mathematics without following a given example.

Before the teaching interviews, Lisa agreed to the statement, "Students should have opportunities to experience manipulating materials in the mathematics classroom before teachers introduce mathematics vocabulary," but did not think it was *necessary*. She stated, "Yes, students should have the opportunity to use manipulatives now and then, but it is not essential. Additionally, I hesitated to say that I strongly agree because I also found it a little difficult to imagine that students could work with materials without knowing the terms of what it was they were looking for." However, after participating in the teaching interviews, she realized that knowing the vocabulary is not critical when trying to find a solution or an explanation/proof. What is important is the process of problem solving and conceptual understanding.

Implications for Preparing Mathematics Teachers

This study was conducted within the course "Learning Mathematics with Technology" (MAE 3651). The results of the study seem to suggest that a course like MAE 3651 be necessary for preservice mathematics teachers (at least currently). First, preservice teachers should understand the mathematics they will be teaching at and beyond the level expected of their future students (MAA, 1991). To the study of geometry, judging, constructing, and communicating mathematically appropriate arguments remain central (NCTM, 2000). Therefore, preservice teachers should be provided with opportunities to develop their mathematical reasoning and proof abilities. In terms of the van Hiele model of geometric thinking, they should reach at least level 4 (ideally level 5). A course like MAE 3651 can assist them towards that goal. Second, geometry has always been a neglected area in mathematics education (NCTM, 1991). A majority of the preservice teachers did not get chances to seriously learn geometry in high school where the focus was on procedures and finding answers. Students like Lisa have stronger geometric background than most of their peers. Even so, however, they lack the necessary knowledge and skills in developing proofs. Students like Fred lack both geometric intuition and conceptual understanding. Therefore, most of the preservice teachers are not ready for *College Geometry*, which is a required course of most teacher preparation programs in the nation. In that course, formal proofs are a norm and students reason about mathematical systems. Students at a thinking level lower than van Hiele level 4 are not able to make sense of the formal aspects of deduction. This explains why it was difficult for most preservice teachers to even pass that course. With a course like MAE 3651 as its preparation, the situation can be fundamentally improved. Third, preservice teachers often have little exposure to the type of investigative mathematics that lies at the heart of NCTM Standards (2000). In a course like MAE 3651, they are required to learn mathematics through hands-on and minds-on investigations and explorations. Finally, dynamic geometry in general, and GSP in particular, is one of the best examples of technology that can be used to support innovative learning and teaching of mathematics. Teacher preparation programs should recognize the need to increase future teachers' knowledge of geometry, help

them develop their own mathematical power, and expose them to innovative methods of mathematics learning and teaching through the use of dynamic geometry software, especially GSP.

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